Phase 13B – ψ Unification & Field-Theoretic Embeddings  
Part 2: Renormalization Group Flows and Fixed Points

Goal  
I aim to analyze how ψ couplings evolve across energy scales using the Renormalization Group (RG) framework. This step determines the stability, universality, and possible fixed points of ψ-gravity when embedded into an Effective Field Theory (EFT). The desert analogy will be extended: RG flows represent shifting dune patterns as the desert floor ψ reshapes under different scale winds.

Effective Action from Part 1  
Recall the compact ψ-EFT action:

Plain text:  
S = ∫ d⁴x √(-g) [ 1/2(∂μψ)(∂^μψ) − αψ² − βψ⁴ + λψ∇²(space+current²) + gψψφ̄φ − 1/4 FμνF^μν − (κ/4)ψFμνF^μν ]

Renormalization Group (RG) Idea  
The couplings vary with scale . Define β-functions:

Plain text:  
βᵢ(gᵢ) = μ d(gᵢ)/dμ

Fixed points occur where .

ψ Self-Couplings  
For the ψ potential :

Mass parameter flow ():

Plain text:  
βα = c1 α + c2 β

Quartic coupling flow ():

Plain text:  
ββ = c3 β² + c4 gψ² − c5 κ²

Constants arise from loop integrals in EFT (determined by AI symbolic integration).

ψ–Matter Yukawa Coupling  
Yukawa-like interaction: .  
The flow is:

Plain text:  
βgψ = a1 gψ³ − a2 gψ β

This indicates ψ–matter strength grows or diminishes depending on ψ quartic self-coupling.

ψ–Gauge Coupling  
For coupling κ:

Plain text:  
βκ = b1 κ g² − b2 κ²

Here is the background gauge coupling. ψ thus reshapes effective gauge strengths depending on κ flow.

ψ–Curvature Coupling  
The Laplacian coupling λ controls desert shaping:

Plain text:  
βλ = d1 λ + d2 κ λ

This indicates curvature and gauge backreaction influence ψ’s effective shaping.

Fixed Point Structures  
Solve for couplings where all β = 0:

* Gaussian fixed point: all couplings → 0.
  + ψ decouples, reducing to free scalar field.
* Interacting fixed point: balance between ψ quartic, Yukawa, and gauge couplings.
  + ψ mediates nontrivial unification regime.
* Infrared desert fixed point:
  + small but positive.
  + ψ provides stable dune structure at large scales.

Desert Analogy of RG Flows

* RG scale = wind intensity shaping dunes.
* Couplings = dune heights shifting as ψ resettles.
* Fixed points = equilibrium dune formations where reshaping halts.

At high energy (UV), dunes shift chaotically (unstable ψ couplings).  
At low energy (IR), dunes stabilize (ψ fixed points).

Python Symbolic RG Flow Prototype

# simulations/phase13B\_part2\_rg\_flows.py  
import sympy as sp  
  
# Define couplings and scale  
alpha, beta, gpsi, kappa, lam, mu = sp.symbols('alpha beta gpsi kappa lam mu')  
c1, c2, c3, c4, c5 = sp.symbols('c1 c2 c3 c4 c5')  
a1, a2 = sp.symbols('a1 a2')  
b1, b2, g = sp.symbols('b1 b2 g')  
d1, d2 = sp.symbols('d1 d2')  
  
# Beta functions (RG flows)  
beta\_alpha = c1\*alpha + c2\*beta  
beta\_beta = c3\*beta\*\*2 + c4\*gpsi\*\*2 - c5\*kappa\*\*2  
beta\_gpsi = a1\*gpsi\*\*3 - a2\*gpsi\*beta  
beta\_kappa = b1\*kappa\*g\*\*2 - b2\*kappa\*\*2  
beta\_lam = d1\*lam + d2\*kappa\*lam  
  
print("β-functions:")  
print("βα =", beta\_alpha)  
print("ββ =", beta\_beta)  
print("βgψ =", beta\_gpsi)  
print("βκ =", beta\_kappa)  
print("βλ =", beta\_lam)  
  
# Solve fixed points (symbolic)  
fixed\_points = sp.solve([  
 sp.Eq(beta\_alpha, 0),  
 sp.Eq(beta\_beta, 0),  
 sp.Eq(beta\_gpsi, 0),  
 sp.Eq(beta\_kappa, 0),  
 sp.Eq(beta\_lam, 0)  
], (alpha, beta, gpsi, kappa, lam), dict=True)  
  
print("\nFixed points:")  
for fp in fixed\_points:  
 print(fp)